Introduction to Polynomials

ACTIVITY 14

Postal Service
Lesson 14-1 Polynomials

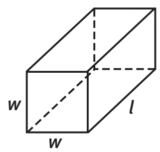
My Notes

Learning Targets:

- Write a third-degree equation that represents a real-world situation.
- Graph a portion of this equation and evaluate the meaning of a relative maximum.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note Taking, Think-Pair-Share

The United States Postal Service will not accept rectangular packages if the perimeter of one end of the package plus the length of the package is greater than 130 in. Consider a rectangular package with square ends as shown in the figure.



1. Assume that the perimeter of one end of the package plus the length of the package equals the maximum 130 in. Complete the table with some possible measurements for the length and width of the package. Then find the corresponding volume of each package.

   Some possible values are shown in the table below.

Width (in.)	Length (in.)	Volume (in.^{3})
10	90	9000
15	70	15,750
20	50	20,000
25	30	18,750
30	10	9000

2. Give an estimate for the largest possible volume of an acceptable United States Postal Service rectangular package with square ends.

   Answers will vary. Estimates should be between 20,000 and 22,000 cubic inches.

Common Core State Standards for Activity 14

HSA-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
HSA-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
HSA-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
HSF-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

ACTIVITY 14
Investigative

Activity Standards Focus

In Activity 14, students are introduced to polynomial functions by writing and graphing a third-degree equation that represents a real-world situation. Then students identify the relative minimums and maximums of third-degree equations and examine the end behavior of polynomial functions. Finally, students determine whether functions are even or odd, using algebraic and geometric techniques.

Students' work with second-degree functions and their graphs, as well as their introduction to the concepts of minimum and maximum, should help students successfully engage with this activity. To help solidify the definition of polynomial functions for students, present examples of functions that students have worked with—linear, quadratic, and exponential—and have students explain whether or not each function is a polynomial function.

Lesson 14-1

PLAN

Pacing: 1 class period

Chunking the Lesson

#1 #2 #3
#4–5 #6–7
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Ask students to find the domain and range of each function.

1. $f(x) = 3x^2 - 9$ [*domain:* $\{x|x \in \mathbb{R}\}$; *range:* $\{f(x)|f(x) \geq -9\}$]
2. $f(x) = -3^x$ [*domain:* $\{x|x \in \mathbb{R}\}$; *range:* $\{f(x)|f(x) < 0\}$]

Discuss with students what the graph of each function looks like and how the graph can be used to find the domain and range.

1 Activating Prior Knowledge Remind students that when calculating the volume, they will need to use the width as a factor twice. Since the package has square ends, both dimensions of the ends are labeled *width*; in a rectangular package, one would have been labeled *height*.

2 Debriefing The purpose of Item 2 is to have students realize that there will be a maximum value. The accuracy of their estimates may vary, and any reasonable estimate should be accepted.

3 Create Representations, Think-Pair-Share After students have answered the question, have them apply the Distributive Property to rewrite the function: $V(w) = -4w^3 + 130w^2$. Pair students to have them share their results. Point out that this function is an example of a cubic function.

4–5 Create Representations Naturally, the width of the box must be greater than 0 inches. If the maximum of 130 inches is divided by 4, the sides of the end of the box, or width, would be 32.5 inches. However, the length of the box must also be included in the 130 inches, so the width must be less than 32.5 inches. Item 4 is designed to have students focus only on the first-quadrant values of the function at this time. Students will explore the function in greater detail later in this lesson.

Once students have a domain and range, they can label the axes and then determine the value for different values of the domain to sketch the graph.

Technology Tip

Students can graph the function using the table of values they have created for Item 1.

For additional technology resources, visit SpringBoard Digital.

CONNECT TO AP

In calculus, students should be prepared to write functions from physical situations with relative ease. You can provide additional practice with different geometric situations. For example, give students a right triangle with one leg labeled x and the other leg labeled $10 - x$. Tell students that the sum of the legs is 10. Make sure to ask why the horizontal leg is labeled $10 - x$. Then ask students to write a function for the area of the triangle. $\left[A(x) = \frac{1}{2}x(10 - x)\right]$

Then assign a similar problem without any variable labels on the diagram. Finally, assign a similar problem, giving only a written description.

My Notes

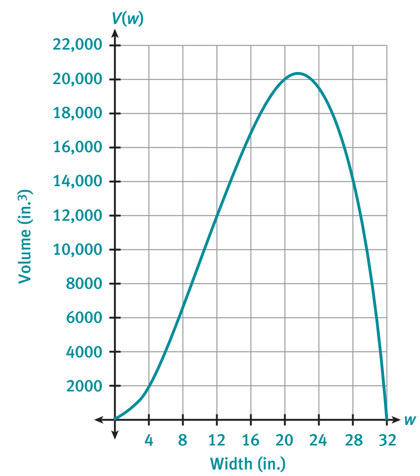
CONNECT TO AP

In calculus, you must be able to model a written description of a physical situation with a function.

3. **Model with mathematics.** Use the package described in Item 1.
   a. Write an expression for ℓ, the length of the package, in terms of w, the width of the square ends of the package.
      $\ell = 130 - 4w$
   b. Write the volume of the package V as a function of w, the width of the square ends of the package.
      $V(w) = (130 - 4w)w^2$
   c. Justify your answer by explaining what each part of your equation represents.
      w^2 represents the square area of the end of the package; $130 - 4w$ represents the length of the package; multiply to get the volume of the package.

4. Consider the smallest and largest possible values for w that make sense for the function you wrote in Item 3b. Give the domain of the function as a model of the volume of the postal package. Express the domain as an inequality, in interval notation, and in set notation.
   $0 < w < 32.5$; $(0, 32.5)$; $\{w \mid w \in \Re, 0 < w < 32.5\}$

5. Sketch a graph of the function in Item 3b over the domain that you found in Item 4. Include the scale on each axis.



Common Core State Standards for Activity 14 *(continued)*

HSF-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Lesson 14-1
Polynomials

6. **Use appropriate tools strategically.** Use a graphing calculator to find the coordinates of the maximum point of the function that you graphed in Item 5.
   (21.667, 20,342.593)

7. What information do the coordinates of the maximum point of the function found in Item 6 provide with respect to an acceptable United States Postal Service package with square ends?
   The coordinates indicate that the maximum possible volume of a package of the type described is 20,342.593 in.^{3} and that this occurs when the package has a width of 21.667 inches.

My Notes

CONNECT TO TECHNOLOGY

Graphing calculators will allow you to find the maximum and minimum of functions in the graphing window.

CONNECT TO AP

In calculus, you will learn about the derivative of a function, which can be used to find the maximum and minimum values of a function.

Check Your Understanding

8. Explain why the function $V(w)$ that you used in this lesson is a third-degree equation.
9. Explain why the value of w cannot equal 0 in this situation.
10. Explain why the value of w must be strictly less than 32.5 in this situation.
11. In this situation, is it possible for the range of the function to be all real numbers? Why or why not?
12. **Critique the reasoning of others.** Another method of shipping at the Post Office allows for the perimeter of one end of a box plus the length of that box to be no greater than 165 inches. Sarena wants to ship a box whose height is twice the width using this method. She says the formula for the volume of such a box is $V(w) = (165 - 6w)2w^2$. Her sister, Monique, says the formula is $V(w) = (165 - w)w^2$. Who is right? Justify your response.

ACTIVITY 14 Continued

6–7 Note Taking, Debriefing, Quickwrite For Item 6, students can find the maximum or minimum of a function on the TI-84 by choosing maximum or minimum in the Calculate menu, which is accessed by pressing 2ND followed by TRACE. For some students, writing this process in their notes will be helpful as they progress through the course.

Students should consult their manuals if they are using any other calculator model. For additional technology resources, visit SpringBoard Digital.

Once students have found the coordinates of the maximum point, have students take a few minutes to write a response to Item 7. You can also ask them what other points on the graph represent, starting with a specific point such as (20, 50,000).

Check Your Understanding

Debrief students' answers to these items to ensure that they understand concepts related to the maximum value for this function. Students should be able to explain why the domain and range are restricted.

Answers

8. The product of 4w and w^2 is $4w^3$.
9. If $w = 0$, the width and volume of the box would be 0, neither of which makes sense.
10. If $w = 32.5$, the length and volume of the box would be 0; if $w > 32.5$, the length of the box would be negative; neither situation makes sense.
11. No; since the domain is restricted, the range will also be restricted.
12. Sarena is correct. If the dimensions of the end of the box are w and $2w$, then the perimeter of the end is $6w$. This means the length of the box is $165 - 6w$. To find the volume, multiply the three dimensions to arrive at Sarena's formula.

ASSESS

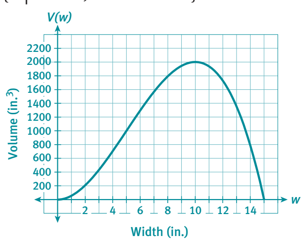
Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 14-1 PRACTICE

13. $0 < w < 15$; $(0, 15)$; $\{w | w \in \Re, 0 < w < 15\}$

14.



15. (10.0, 2000)

16. 10 in.

17. **a.** $h = 60 - 2r$

b. $V = \pi r^2(60 - 2r)$

c. 20 cm

d. 25,132.741 cm^3

ADAPT

Check students' answers to the Lesson Practice to ensure that they can draw a graph and find the maximum values. If students cannot proficiently use a calculator, they will struggle with the remainder of the content in this activity.

LESSON 14-1 PRACTICE

13. The volume of a rectangular box is given by the function $V(w) = (60 - 4w)w^2$. What is a reasonable domain for the function in this situation? Express the domain as an inequality, in interval notation, and in set notation.

14. Sketch a graph of the function in Item 13 over the domain that you found. Include the scale on each axis.

15. Use a graphing calculator to find the coordinates of the maximum point of the function given in Item 13.

16. What is the width of the box, in inches, that produces the maximum volume?

17. **Reason abstractly.** An architect uses a cylindrical tube to ship blueprints to a client. The height of the tube plus twice its radius must be less than 60 cm.

a. Write an expression for h, the height of the tube, in terms of r, the radius of the tube.

b. Write an expression for V, the volume of the tube, in terms of r, the radius of the tube.

c. Find the radius that produces the maximum volume.

d. Find the maximum volume of the tube.

Lesson 14-2
Some Attributes of Polynomial Functions

My Notes

Learning Targets:
- Sketch the graphs of cubic functions.
- Identify the end behavior of polynomial functions.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Marking the Text, Create Representations, Predict and Confirm

When using a function to model a given situation, such as the acceptable United States Postal Service package, the reasonable domain may be only a portion of the domain of real numbers. Moving beyond the specific situation, you can examine the *polynomial function* across the domain of the real numbers.

A ***polynomial function*** in one variable is a function that can be written in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where n is a nonnegative integer, the coefficients $a_0, a_1, \ldots a_n$ are real numbers, and $a_n \neq 0$. The highest power, n, is the ***degree*** of the polynomial function.

A polynomial is in ***standard form*** when all like terms have been combined, and the terms are written in descending order by exponent.

Leading coefficient — Degree of the polynomial

$$7x^5 + 2x^2 - 3$$

term — constant

Various attributes of the graph of a polynomial can be predicted by its equation. Here are some examples:

- the constant term is the y-intercept of the graph;
- the degree tells the maximum number of x-intercepts the graph of a polynomial can have; and
- the degree of the polynomial gives you information about the shape of the graph at its ends.

MATH TERMS

Some common types of **polynomial functions** are listed in the table. You are already familiar with some of these.

Polynomial functions are named by the **degree** of the function.

Degree	Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic

Lesson 14-2

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–3 #4–6
Check Your Understanding
#12–13 #14–20
Check Your Understanding
Lesson Practice

TEACH

Bell-Ringer Activity

Name the type of function.

1. $5x^2 + 7x - 18$ [*quadratic*]
2. $7x - 18$ [*linear*]
3. $2^x - 1$ [*exponential*]

Developing Math Language

Writing a polynomial in standard form makes it easy to identify the number of terms and degree of the polynomial. Use the definition of a polynomial function to help students understand why the following are not polynomial functions:

- $2a^{-3} + a^2 + 1$
- $x^{\frac{1}{2}} - x + 14$
- $x^2 + y^2$

Introduction Close Reading, Marking the Text Polynomial functions are named by their degree. Ask students to give examples of constant, linear, quadratic, cubic, and quartic polynomials.

1–3 Create Representations, Vocabulary Organizer In Item 1, students move beyond the context of the acceptable rectangular package and examine the graph of the cubic function from Lesson 14-1 in all four quadrants.

For Item 2, students can estimate the relative maximum and relative minimum by inspection of the graph. They can find the exact values by using a graphing calculator.

In Item 3, students can find the x- and y-intercepts by factoring the cubic function or by using a graphing calculator. The function is simple enough to factor and leads to a discussion that prepares students for Lesson 14-3.

This would be an appropriate time to investigate the number of x-intercepts that a cubic function may have. This function has two x-intercepts. Ask students to use their graphing calculators to explore whether a cubic function can have fewer than or more than two x-intercepts [*a cubic function can have 1, 2, or 3 x-intercepts*].

As students respond to questions or discuss possible solutions to problems, monitor their use of new terms and descriptions of applying math concepts to ensure their understanding and ability to use language correctly and precisely.

ELL Support

Students may struggle to understand the meanings of *maximum*, *minimum*, *relative maximum*, and *relative minimum* in this context. First address the common meanings of the words *maximum* and *minimum* and how the words relate to and are different from the words *greatest* and *least*. Discuss different meanings for the word *relative* and lead students to understand the meaning in this context.

4–6 Create Representations, Predict and Confirm Ask students to predict what the graph of the function in Item 4 will look like, and what it won't look like (a straight line or a parabola, for instance). After students have graphed the function, have them describe how this graph is similar to and different from the graph in Item 1.

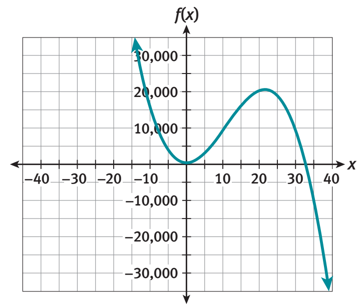
ACTIVITY 14
continued

My Notes

Lesson 14-2
Some Attributes of Polynomial Functions

1. Write a polynomial function $f(x)$ defined over the set of real numbers in standard form such that it has the same function rule as $V(w)$, the rule you found in Item 3b of the previous lesson for the volume of the rectangular box. Sketch a graph of the function.

$f(x) = -4x^3 + 130x^2$



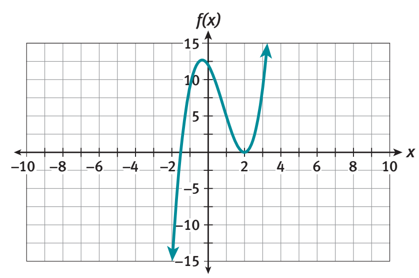
2. Name any ***relative maximum*** values and ***relative minimum*** values of the function $f(x)$ in Item 1.

The relative minimum is zero. The relative maximum is about 20,342.593.

3. Name any x- or y-intercepts of the function $f(x) = -4x^3 + 130x^2$.

The x-intercepts are 0 and 32.5. The y-intercept is 0.

4. Model with mathematics. Use a graphing calculator to sketch a graph of the cubic function $f(x) = 2x^3 - 5x^2 - 4x + 12$.



MATH TERMS

A function value $f(a)$ is called a **relative maximum** of f if there is an interval around a where, for any x in the interval, $f(a) \geq f(x)$.

A function value $f(a)$ is called a **relative minimum** of f if there is an interval around a where, for any x in the interval, $f(a) \leq f(x)$.

MATH TIP

Think of a *relative minimum* of a graph as being the bottom of a hill and the *relative maximum* as the top of a hill.

TECHNOLOGY TIP

When the coefficients of an equation are relatively small, begin with a standard 10-by-10 viewing window, and then adjust the window if necessary.

Universal Access

Students may mistakenly use values from the x-axis as the relative maximum and minimum. For instance, they may think that the relative maximum of the function in Item 1 is about 21. Thinking of relative maximum and minimum as measures of the "height" of the function may help students associate (relative) maximum and (relative) minimum with values on the y-axis.

Lesson 14-2
Some Attributes of Polynomial Functions

ACTIVITY 14
continued

My Notes

5. Name any relative maximum values and relative minimum values of the function $f(x)$ in Item 4.
The relative minimum is zero. The relative maximum is about 12.704.

6. Name any x- or y-intercepts of the function in Item 4.
The x-intercepts are -1.5 and 2. The y-intercept is 12.

Check Your Understanding

7. Decide if the function $f(x) = 7x - 2 + x^2 - 4x^3$ is a polynomial. If it is, write the function in standard form and then state the degree and leading coefficient.

8. **Construct viable arguments.** Explain why $f(x) = 2x + 5 - \frac{1}{x}$ is not a polynomial.

9. Use a graphing calculator to sketch a graph of the cubic function $f(x) = x^3 + x^2 - 4x - 2$.

10. Use a graphing calculator to determine how many x-intercepts the graph of $f(x) = x^3 + x^2 - 4x + 5$ has.

11. **Use appropriate tools strategically.** Use the graphs you have sketched in this lesson to speculate about the minimum number of times a cubic function must cross the x-axis and the maximum number of times it can cross the x-axis.

The ***end behavior*** of a graph is the appearance of the graph on the extreme right and left ends of the x-axis. That is, you look to see what happens to y as x approaches $-\infty$ and ∞.

Examine your graph from Item 1. To describe the end behavior of the graph, you would say: The left side of the graph increases (points upward) continuously and the right side of the graph decreases (points downward) continuously. You can also use mathematical notation, called *arrow notation*, to describe end behavior. For this graph you would write:
As $x \to -\infty$, $y \to \infty$, and as $x \to \infty$, $y \to -\infty$.

12. Examine your graph from Item 4. Describe the end behavior of the graph in words and by using arrow notation.
The left side of the graph decreases continuously and the right side increases continuously; as $x \to -\infty$, $y \to -\infty$, and as $x \to \infty$, $y \to \infty$.

MATH TERMS
End behavior describes what happens to a graph at the extreme ends of the x-axis, as x approaches $-\infty$ and ∞.

MATH TIP
Recall that the phrase *approaches positive infinity* or *approaches* ∞ means "increases continuously," and that approaches *negative infinity* or *approaches* $-\infty$ means "decreases continuously."

Values that increase or decrease continuously, or without stopping, are said to increase or decrease *without bound.*

ACTIVITY 14 Continued

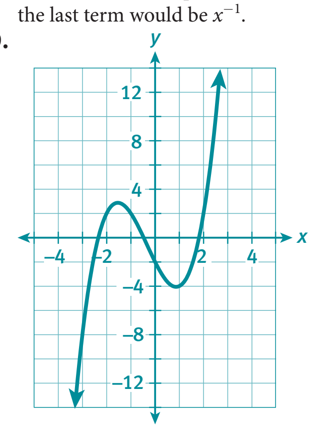
4–6 (continued) After students answer Items 5 and 6, ask them if and how the x- and y-intercepts are related to the relative maximums and minimums [*x-intercepts lie between a relative maximum and a relative minimum but not necessarily between each relative maximum and minimum; y-intercepts have no relationship with extrema*].

Check Your Understanding
Debrief students' answers to these items to ensure that they understand that the exponent of the first term of a polynomial written in standard form equals the degree of the equation. Have students graph the equation in Item 8 and discuss how its shape differs from the polynomial function in Item 9.

Answers
7. Yes; $f(x) = -4x^3 + x^2 + 7x - 2$; degree: 3; leading coefficient: -4

8. A polynomial can only have nonnegative integer exponents. When written in exponential form, the last term would be x^{-1}.

9.



10. one

11. minimum: 1; maximum: 3

12–13 Create Representations, Vocabulary Organizer The symbols $-\infty$ and ∞ have a special meaning in the context of limits. Therefore, remind students that, in this instance, they should remember that the phrase *approaches positive infinity* means "increases without bound," and that *approaches negative infinity* means "decreases without bound."

12–13 (continued) These two items provide the first opportunity for students to consider the end behavior of polynomial functions. They will investigate this concept in more detail in the items that follow.

14–15 Create Representations, Discussion Groups As an alternative to sketching the graphs in Item 14, have students play a matching game. Allow students to work in groups. Half of the group has cards showing the equations of the graphs, while the other half has cards displaying the graphs. Have students work together to match equations with graphs. Ask students to justify their reasoning and the reasonableness of their solutions. Remind students to use specific details and precise mathematical language in their justifications.

My Notes

MATH TIP

The leading term of a polynomial (which has the greatest power when the polynomial is written in standard form) determines the end behavior. Learning these basic polynomial shapes will help you describe the end behavior of any polynomial.

13. Examine the end behavior of $f(x) = 3x^2 - 6$.

a. As x goes to ∞, what behavior does the function have?

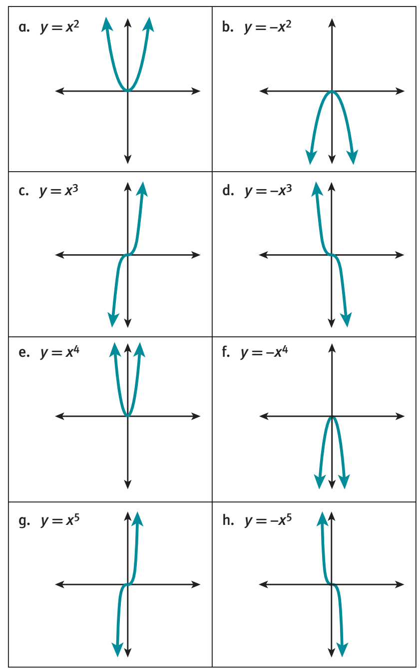
$x \to \infty, y \to \infty$

b. How is the function behaving as x approaches $-\infty$?

$x \to -\infty, y \to \infty$

It is possible to determine the end behavior of a polynomial's graph simply by looking at the degree of the polynomial and the sign of the leading coefficient.

14. Use appropriate tools strategically. Use a graphing calculator to examine the *end behavior* of polynomial functions in general. Sketch each given function on the axes below.



Lesson 14-2
Some Attributes of Polynomial Functions

ACTIVITY 14
continued

My Notes

15. Which of the functions in Item 14 have the same end behavior on the right side of the graph as on the left side?
$y = x^2$, $y = -x^2$, $y = x^4$, and $y = -x^4$

16. Reason quantitatively. What is true about the degree of each of the functions you identified in Item 15?
The degree is even.

17. Make a conjecture about how the degree affects the end behavior of polynomial functions.
Sample answer: Polynomial functions that have even degrees have end behavior in the same direction at both ends of the x-axis, and polynomial functions with odd degrees have end behavior in the opposite direction at the ends of the x-axis.

18. For which of the functions that you identified in Item 15 does the end behavior decrease without bound on both sides of the graph?
$y = -x^2$ and $y = -x^4$

19. What is true about the leading coefficient of each of the functions you identified in Item 18?
The leading coefficient is negative.

20. Express regularity in repeated reasoning. Make a conjecture about how the sign of the leading coefficient affects the end behavior of polynomial functions.
Sample answer: Polynomial functions that have a negative leading coefficient have end behavior in the opposite (vertical) direction as the same polynomial with a positive leading coefficient. Polynomial functions with a positive leading coefficient increase without bound as $x \to \infty$, while polynomial functions with a negative leading coefficient decrease without bound as $x \to \infty$.

16–20 Create Representations To prepare for the next lesson, ask students to describe the symmetry of each of the graphs.

Ask students to describe any patterns they notice as well as any relationship between end behavior and symmetry. In particular, ask:

- What does the graph of a polynomial of even degree look like?
- What does the graph of a polynomial of odd degree look like?
- When the leading coefficient of a polynomial function changes from positive to negative, how does its graph change?

Before students answer the above questions and work through the rest of the items in this activity, have them create a graphic organizer like the one below and fill it in for future reference.

Polynomial Functions			
Degree	Degree: Even or Odd	Sign of Leading Coefficient	End Behavior
2		+	
2		–	
3		+	
3		–	
4		+	
4		–	
5		+	
5		–	
6		+	
6		–	

Check Your Understanding

Debrief students' answers to these items to ensure that they can identify the end behavior of a polynomial function. Have students describe the possible end behaviors for a function and what these behaviors look like on a graph.

Answers

21. As $x \to -\infty$, $y \to -\infty$.
22. Both sides of the graph decrease without bound (or point downward).
23. No; the end behavior is the same on both sides of the graph, so the degree of the polynomial is even.
24. Answers will vary. Any polynomial with an even degree and a positive leading coefficient is acceptable.

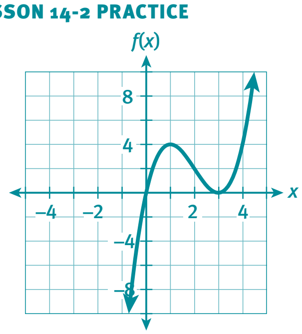
ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 14-2 PRACTICE

25.



26. x-intercepts: 0 and 3; y-intercept: 0; relative maximum: (1, 4); relative minimum: (3, 0)
27. Answers will vary. Check students' work for the correct number of intercepts, the correct relative maximum or minimum, and end behavior in opposite directions.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to identify the end behavior, relative maximums and minimums, and intercepts of the graph of a polynomial function. Have students circle each of these key features on the graphs of these functions.

My Notes

Check Your Understanding

21. Use arrow notation to describe the left-end behavior of a graph that decreases without bound.

22. Describe in words the end behavior of a graph that is described by the following arrow notation: As $x \to \pm\infty$, $y \to -\infty$.

23. **Reason abstractly.** If the end behavior of a graph meets the description in Item 22, is it possible that the graph represents a third-degree polynomial? Explain your answer.

24. Give two examples of a polynomial whose graph increases without bound as x approaches both positive and negative infinity.

LESSON 14-2 PRACTICE

25. Sketch the graph of the polynomial function $f(x) = x^3 - 6x^2 + 9x$.

26. Name any x-intercepts, y-intercepts, relative maximums, and relative minimums for the function in Item 25.

27. **Make sense of problems.** Sketch a graph of any third-degree polynomial function that has three distinct x-intercepts, a relative minimum at $(-6, -4)$, and a relative maximum at $(3, 5)$.

28. Decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.
a. $f(x) = 5x - x^3 + 3x^5 - 2$
b. $f(x) = -\frac{2}{3}x^3 - 8x^4 - 2x + 7$
c. $f(x) = 4^x + 2x^2 + x + 5$

29. Describe the end behavior of each function.
a. $f(x) = x^6 - 2x^3 + 3x^2 + 2$
b. $f(x) = -\frac{2}{3}x^3 - 8x^2 - 2x + 7$

28. a. yes; $f(x) = 3x^5 - x^3 + 5x - 2$; fifth degree; 3
b. yes; $f(x) = -8x^4 - \frac{2}{3}x^3 - 2x + 7$; fourth degree; -8
c. no

29. a. As $x \to \pm\infty$, $f(x) \to \infty$.
b. As $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to \infty$.

My Notes

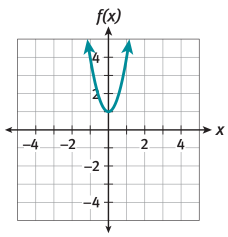
Learning Targets:

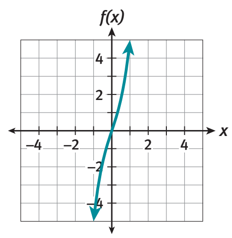
- Recognize even and odd functions given an equation or graph.
- Distinguish between even and odd functions and even-degree and odd-degree functions.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Marking the Text, Create Representations

The graphs of some polynomial functions have special attributes that are determined by the value of the exponents in the polynomial.

1. Graph the functions $f(x) = 3x^2 + 1$ and $f(x) = 2x^3 + 3x$ on the axes.





2. Describe the symmetry of the graph of $f(x) = 3x^2 + 1$.
The graph is symmetric over the *y*-axis.

3. Describe the symmetry of the graph of $f(x) = 2x^3 + 3x$.
The graph is symmetric around the origin.

The function $f(x) = 3x^2 + 1$ is called an ***even function***. Notice that *every* power of x is an even number—there is no x^1 term. This is true for the constant term as well, since you can write a constant term as the coefficient of x^0. Symmetry over the y-axis is an attribute of all even functions.

The function $f(x) = 2x^3 + 3x$ is an ***odd function***. Notice that every power of x is an odd number—there is no x^2 or constant (x^0) term. Symmetry around the origin is an attribute of all odd functions.

MATH TIP

The graph of a function can be symmetric across an axis or other line when the graph forms a mirror image across the line. The graph can be symmetric around a point when rotation of the graph can superimpose the image on the original graph.

MATH TERMS

Algebraically, an **even function** is one in which $f(-x) = f(x)$.

An **odd function** is one in which $f(-x) = -f(x)$.

Lesson 14-3

PLAN

Pacing: 1 class period

Chunking the Lesson

#1–3 #4–5

Check Your Understanding

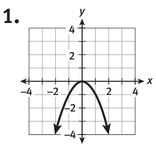
Lesson Practice

TEACH

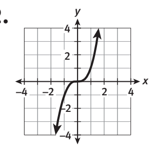
Bell-Ringer Activity

Describe the degree and sign of the leading coefficient of the following graphs. Then describe any symmetry they have.

1.



2.



[*1. even degree; negative; symmetry over the y-axis*] [*2. odd degree; positive; symmetry around the origin*]

Developing Math Language

Help students understand why functions are called *even* and *odd* by stressing the relationship between even and odd functions and even and odd numbers. Students can circle the powers of x (or any variable) in a polynomial function to better understand this.

1–3 Create Representations Have students substitute x with $-x$ in each function to show the symmetry of even and odd functions algebraically.

For $f(x) = 3x^2 + 1$:

$f(-x) = 3(-x)^2 + 1 = 3x^2 + 1 = f(x)$

So, the function is even, and we have just shown that the graph is symmetric over the y-axis.

For $f(x) = 2x^3 + 3x$:

$f(-x) = 2(-x)^3 + 3(-x) = -2x^3 - 3x = -(2x^3 + 3x) = -f(x)$

So, the function is odd and we have just shown that the graph is symmetric around the origin.

4–5 Marking the Text Tell students they can mark the graphs to show the symmetry of the curve and determine whether each function is odd or even.

Check Your Understanding

Debrief students' answers to these items to ensure that they can distinguish even and odd functions. Debrief Item 6 by discussing how a function that is neither even nor odd can still have symmetry in its graph. The symmetry will be across a line other than the y-axis or around a point other than the origin.

Answers

6. Sample answer: The exponents are not all even ($8x$ is odd), nor are they all odd ($4x^2$ is even).
7. No; an even function must have an even degree. The end behavior of a function with an even degree is the same on both ends of the graph, which is not the case here.

ASSESS

Students' answers to Lesson Practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

LESSON 14-3 PRACTICE

8. The function is neither even nor odd. The first two terms have odd powers of x while the last term has an even power (x^0).
9. The function is even because the graph is symmetric over the y-axis.
10. Sample answer: $f(x) = x^3 + x^2$.

ADAPT

Check students' answers to the Lesson Practice to ensure that they understand how to identify even and odd functions. If students give the wrong answer to Item 8, have them graph the function and determine by inspection whether it has the correct symmetry.

My Notes

4. Examine the sketches you made in Item 14 of the previous lesson. Use symmetry to determine which graphs are even functions and which are odd functions. Explain your reasoning.
   Even: $y = x^2$, $y = -x^2$, $y = x^4$, and $y = -x^4$; each of these graphs is symmetric over the y-axis.
   Odd: $y = x^3$, $y = -x^3$, $y = x^5$, and $y = -x^5$; each of these graphs is symmetric around the origin.

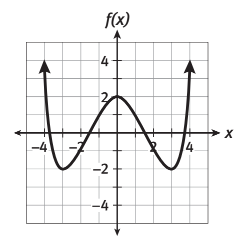
5. **Make use of structure.** Explain how an examination of the equations in Item 14 of the previous lesson supports your answer to Item 4.
   The even functions have only even powers of x while the odd functions have only odd powers of x.

Check Your Understanding

6. Explain why the function $f(x) = 4x^2 + 8x$ is neither even nor odd.
7. For a given polynomial function, as x approaches $-\infty$ the graph increases without bound, and as x approaches ∞ the graph decreases without bound. Is it possible that this function is an even function? Explain your reasoning.

LESSON 14-3 PRACTICE

8. Determine whether the function $f(x) = 2x^5 + 3x^3 + 7$ is even, odd, or neither. Explain your reasoning.
9. Determine whether the function below is even, odd, or neither. Justify your answer.



10. **Attend to precision.** Give an example of a polynomial function that has an odd degree, but is not an odd function.

ACTIVITY 14 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 14-1

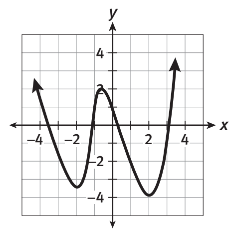
1. The volume of a rectangular box is given by the expression $V = (120 - 6w)w^2$, where w is measured in inches.
   a. What is a reasonable domain for the function in this situation? Express the domain as an inequality, in interval notation, and in set notation.
   b. Sketch a graph of the function over the domain that you found. Include the scale on each axis.
   c. Use a graphing calculator to find the coordinates of the maximum point of the function.
   d. What is the width of the box, in inches, that produces the maximum volume?
2. A cylindrical can is being designed for a new product. The height of the can plus twice its radius must be 45 cm.
   a. Find an equation that represents the volume of the can, given the radius.
   b. Find the radius that yields the maximum volume.
   c. Find the maximum volume of the can.

Lesson 14-2

3. Sketch the graph of the polynomial function $f(x) = -x^3 + 4x^2 - 4x$.
4. Name any x- or y-intercepts of the function $f(x)$ in Item 3.
5. Name any relative maximum values and relative minimum values of the function $f(x)$ in Item 3.

For Items 6–10, decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.

6. $f(x) = 7x^2 - 9x^3 + 3x^7 - 2$
7. $f(x) = 2x^3 + x - 5^x + 9$
8. $f(x) = x^4 + x + 5 - \frac{1}{4}x^3$
9. $f(x) = -0.32x^3 + 0.08x^4 + 5^{x-1} - 3$
10. $f(x) = 3x + 5 + \sqrt{x}$
11. Examine the graph below.

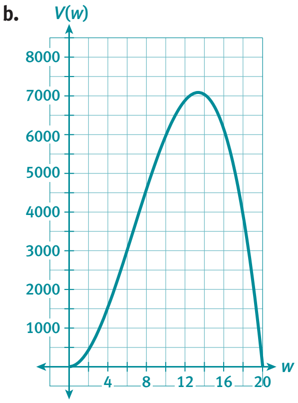


Which of the following statements is NOT true regarding the polynomial whose graph is shown?

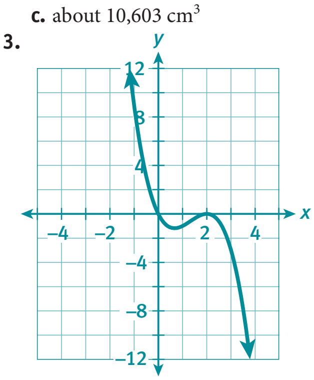
A. The degree of the polynomial is even.
B. The leading coefficient is positive.
C. The function is a second-degree polynomial.
D. As $x \to \pm\infty$, $y \to \infty$.

ACTIVITY PRACTICE

1. **a.** $0 < w < 20$; $(0, 20)$; $\{w | w \in \Re, 0 < w < 20\}$
   b.



   c. (13.333, 7111.111)
   d. about 13.3 inches
2. **a.** $V = -2\pi r^3 + 45\pi r^2$
   b. 15 cm
   c. about 10,603 cm^3
3.



4. x-intercepts: 0 and 2; y-intercept: 0
5. relative maximum value: 0; relative minimum value: -1.185
6. Yes; $f(x) = 3x^7 - 9x^3 + 7x^2 - 2$; seventh degree; 3
7. No.
8. Yes; $f(x) = x^4 - \frac{1}{4}x^3 + x + 5$; fourth degree; 1
9. No.
10. No.
11. C

12. As $x \to \pm\infty$, $f(x) \to \infty$.
13. As $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to \infty$.
14. Polynomials are continuous functions. Since one side of the graph increases without bound and the other side decreases without bound, the graph must cross the x-axis in at least one place.
15. Check students' work.
16. even
17. neither
18. odd
19. B
20. Check students' work.
21. $(-5, 3)$; Since an even function is symmetric over the y-axis, you can reflect the point $(5, 3)$ over the y-axis to get the point $(-5, 3)$.
22. Sharon is correct that the function is a polynomial function and that it has a positive leading coefficient. However, the function is not an even function because it is not symmetric over the y-axis. She is also incorrect about the degree; since the graph crosses the x-axis four times, it must be at least a fourth-degree polynomial.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

For Items 12 and 13, describe the end behavior of each function using arrow notation.

12. $f(x) = x^6 - 2x^3 + 3x^2 + 2$

13. $f(x) = -x^3 + 7x^2 - 11$

14. Use the concept of end behavior to explain why a third-degree polynomial function must have at least one x-intercept.

15. Sketch a graph of any third-degree polynomial function that has exactly one x-intercept, a relative minimum at $(-2, 1)$, and a relative maximum at $(4, 3)$.

Lesson 14-3

For Items 16–28, determine whether each function is even, odd, or neither.

16. $f(x) = 10 + 3x^2$

17. $f(x) = -x^3 + 2x + 5$

18. $f(x) = 6x^5 - 4x$

19. When graphed, which of the following polynomial functions is symmetric about the origin?
A. $f(x) = -x^3 + 2x + 5$
B. $f(x) = x^3 + 8x$
C. $f(x) = -7x^2 + 5$
D. $f(x) = 5x^3 + 3x^2 - 7x + 1$

20. Sketch a graph of an even function whose degree is greater than 2.

21. If $f(x)$ is an even function and passes through the point $(5, 3)$, what other point must lie on the graph of the function? Explain your reasoning.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

22. Sharon described the function graphed below as follows:
- It is a polynomial function.
- It is an even function.
- It has a positive leading coefficient.
- The degree n could be any even number greater than or equal to 2.

Critique Sharon's description. If you disagree with any of her statements, provide specific reasons as to why.